

# **A “Bids and Offers” Approach to Package Bidding**

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## **Overview**

The FCC has an expressed need for an auction procedure that would permit bidders to submit bids on packages of licenses. This paper discusses some of the issues that such a procedure must address, and proposes a particular “bids and offers” procedure (where offers are provisional, temporary bids) designed to deal with those issues.<sup>1</sup>

## **The Exposure Problem**

In the simultaneous multi-round (SMR) auction procedure currently used by the FCC to sell spectrum licenses, each submitted bid is for a single license. When licenses have complementary value—when for some bidders, a package of licenses is worth more than the sum of the values of the individual licenses—this restriction of bids to individual licenses gives rise to the “exposure problem”: A bidder seeking to acquire a package of complementary licenses might find itself the high bidder on some licenses, at prices above their individual values, and then see the prices of other licenses needed to form the package rise to a level that brings the total package price above its value to the bidder. (The simplest example involves a bidder who assigns no value to licenses A and B separately, and positive value to the pair. If the bidder ever bids on one of the licenses, that bid might subsequently never be topped, while the other license gets bid up to a price exceeding the bidder’s valuation for the pair.)

In an attempt to deal with the exposure problem, the FCC has permitted bid withdrawals in many of its auctions. However, to avoid facing frivolous or disruptive bids and subsequent withdrawals, the FCC has imposed bid-withdrawal penalties upon bidders. The exposure to penalties can, in itself, deter bidders seeking to acquire large packages.

## **“Only If” Bids**

An alternative approach would be to allow bidders to submit “only if” bids. In the example above, the bidder could submit bids for license A which stay on the table only if the bidder eventually wins license B, and bids for B which stay only if A is won. (Such bids are, in a sense, precommitments to bid withdrawal under certain circumstances.)

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<sup>1</sup> The package-bidding auction procedure proposed here contains pieces of previous proposals made by numerous contributors to the theory and practice of auctions. Questions, comments, criticisms, suggestions, and revisions are welcome.

## **Package Bids**

One way to facilitate “only if” bidding is to allow bidders to submit bids on packages of licenses. A package bid on the set of licenses  $\{A,B\}$  would express a willingness to pay only if the bidder receives both licenses at the end of the auction.

Indeed, if bidders are free to bid on packages, other types of “only if” problems are also solved. For example, consider the “hub problem,” in which a bidder seeks a license (A) covering a metropolitan area, and desires an adjacent license (B) covering a rural area only if the metropolitan license is won. The bidder could simply submit two package bids: one on A alone, and another on the package  $\{A,B\}$ .

Of course, if package bids are allowed, the FCC must determine how the licenses will finally be allocated: Holding individual bids on licenses A, B, and C as well as package bids for  $\{A,B\}$ ,  $\{A,C\}$ , and  $\{B,C\}$  (and possible even for  $\{A,B,C\}$ ), how should the FCC determine the winning collection of bids? The “obvious” approach is to determine the collection of non-overlapping packages that maximizes the sum of all associated bids, and award the licenses accordingly.

However, this introduces new problems.

## **The Exclusive-Or Problem**

In auctions already held by the FCC, it has been clear that bidders have at times held business plans (or faced budget constraints) which made some licenses substitutes for others. For example, an auction might offer two licenses (A and B) covering equal bandwidth over the same geographic region, and a bidder might consider either license sufficient to its needs, i.e., once either license is held, the other is of no direct value to the bidder. The bidder would want to bid in a manner which eventually obtains either A or B (if the price is acceptable), but not both. Equivalently, the bidder would want to be able to submit “only if” bids of the form “A only if I don’t get B”, and “B only if I don’t get A”.

Package bidding offers a direct solution to this problem if bidders are allowed to bid on all packages. Rather than determining the collection of packages that maximizes the sum of all associated bids, the FCC could restrict itself to determining the partition of licenses among bidders that is sum-of-bids maximizing when the set of licenses assigned to a single bidder must be viewed as a package. In the example above, the bidder could submit a bid for A, a bid for B, and a bid for  $\{A,B\}$  which is the smaller of the two individual bids. If the FCC then assigned both A and B to this bidder, the bid that would count (towards the sum-of-bids) would be the  $\{A,B\}$  bid.

## Practical Matters

In theoretical terms, allowing bidders to bid on all packages would solve the exclusive-or problem. In practical terms, it might well impose too high a bid-submission cost on bidders, or a computational burden on the FCC. (A bidder merely seeking any one license alone from a group of ten would need to submit over a thousand package bids.) For these reasons, the FCC might choose to restrict the list of multi-license packages on which bids may be submitted. Certainly, it would be advantageous for the FCC to have available for its use an auction procedure that permits limited package bidding.

As well, the FCC has shown a reluctance to force bidders to face the strategic challenges inherent in one-shot sealed-bid auctions. Throughout the remainder of this paper, we focus on package bidding in the context of multi-round auctions.

Bidders already have some familiarity with the SMR auction procedure. A desirable property of any new procedure which allows the submission of package bids would be that, in the case where bidders are restricted to single-license bids only (i.e., to singleton packages), the new procedure closely resembles the SMR procedure.

## The Threshold Problem

Consider an example in which bidder 1 seeks only license A (and would be willing to bid up to 14 for it), and bidder 2 seeks only license B (and would also be willing to bid up to 14). Each currently has bid 5 for the license it wants, and another bidder (3) has bid 20 for {A,B}. What should the first two bidders do? Somehow they must work together, each raising its bid enough that the two individual bids together rise by at least  $20 - (5+5) = 10$ . The need to raise their bids (in aggregate) by 10 forms a “threshold” they must cross together: Neither can do this alone.

In this simple example, an outside observer might consider it “fair” that bidders 1 and 2 raise their bids by the same amount. But each might be tempted to hold back, hoping the other will eventually bear more than half of the threshold burden. After all, if bidder 1 eventually raises its bid to 12, this lowers the amount bidder 2 must bid to help beat bidder 3. Furthermore, if bidder 2 is truly committed to not raising its bid above 8, it is rational for bidder 1 to eventually accept more than half the burden.

In practice, of course, asymmetries between the bidders’ valuations and between the natures of individual licenses make it impossible even to determine “fair” shares.

Any dynamic bidding process involving package bids must, in order to achieve a relatively efficient allocation of licenses, facilitate the “bargaining” that bidders must engage in at times in order to overcome the threshold problem.

The “bids and offers” proposal laid out in the following sections approaches the threshold problem of the above example by noting that, in order to beat the {A,B} bid, at least one of the two single-license bidders must be willing to bear at least half of the threshold burden.

## The “Bids and Offers” Procedure

### Rounds

The dynamic auction procedure to be described proceeds in *rounds* of fixed duration. A round begins with a public posting of the currently-high bids on all (allowed) packages (including singleton packages, i.e., bids on single licenses). As well, the *prospectively-winning collection* of bids and offers is announced: This is the collection of non-overlapping bids and offers that maximizes the total associated bid amount. More than one package (i.e., bids or offers on disjoint packages) from the same bidder may be included in this collection. (The “exclusive-or” problem is therefore an issue to be dealt with.) Finally, the minimum acceptable increments to all bids are posted. In the discussion that follows, it is assumed that the minimum increments are a specified percentage of the current high bids, and that the same percentage determines minimum offer increments.

### Bids and Offers

Bidders are allowed to make two different types of submissions. The first type, which is simply called a *bid*, involves submitting a price for a package that exceeds the “minimum acceptable bid” level (defined below) for that package. When one or more bids on a package are processed, the highest one (with ties broken in favor of the earliest bid received) becomes the new currently-high bid for the package. All other bids on that package, including the previous high bid, are removed from the system. A bid that is declared a currently-high bid cannot be withdrawn. It persists across rounds of the auction until a new, higher bid (or collection of bids) replaces it.

The other type of submission, called an *offer*, also involves submitting a price for a package. However, offers must meet a (typically) more stringent minimum requirement (described below) than must bids. When one or more offers on a package are processed, the highest is made public (and all other offers on that package are removed from the system). If, during the round, an offer becomes a member of a newly-announced prospectively-winning collection, it continues to remain active (even across rounds) until the announcement of a subsequent prospectively-winning collection of which it is not a member. At the end of a round, all offers that are not part of the current prospectively-winning collection are removed from the system.

## **The Round Structure**

At the beginning of a round, the public postings described above take place.

Each round then moves into a *quiet period* of fixed duration, during which bids and offers may be submitted. At the end of the quiet period, all bids and offers are processed. The prospectively-winning collection of bids and offers is announced, together with all active (i.e., currently-high) bids and offers.

A continuous-time *open period* now begins, and continues until the scheduled closing “quiet” period. Bids and offers can be submitted at any time during the open period, and (if they become active) are made public as soon as possible. Whenever a new collection of bids and offers tops the current prospectively-winning collection, the new prospectively-winning collection is announced.

At the end of the round, a second *quiet period* takes place, handled just as was the first.

After a brief breather, the next round begins.

## **Ending the Auction**

If, through the course of an entire round, no new prospectively-winning collection emerges, it is announced that the following round might be the last. If the following round yields a new prospectively-winning collection, the auction reverts to its normal mode and continues. However, if no new prospectively-winning collection emerges, the auction ends. [An acceptable alternative would be to end the auction immediately after the first round that fails to yield a new prospectively-winning collection. In practical terms, this would put substantial pressure on the bidders at the start of every round, which would be relieved when a new prospectively-winning collection is announced. It is not clear whether this pressure-relief-pressure-relief cycle is desirable.]

## **Bid Removal**

If a currently-high bid on a package is topped by a collection of bids on non-overlapping subpackages, that bid is removed from the system. The rationale is that, since bids cannot be withdrawn, this bid will never again be part of a prospectively-winning collection.

## Minimum Bid Requirements

In order to determine the minimum acceptable bid a bidder is allowed to place on a target package, the steps below are carried out:

- (1) All currently-high bids (not offers) submitted by other bidders are raised (for this calculation only) by their minimum increments. The total-maximizing collection of bids on non-overlapping subpackages (this collection might include some of the bidder's own bids) of the target package is determined, and the associated total bid amount is computed.
- (2) If the total-maximizing collection determined in (1) consists solely of bids submitted by this bidder, the minimum bid increment associated with the smallest of those bids is added to the total amount determined in (1).

The purpose in not raising the bidder's own bids in Step 1 is to keep bidders from having to worry that their currently-high bids on subpackages might work against them. Step 2 is purely cosmetic: Its primary purpose is to stop bidders from submitting equivalent bids twice.

## Examples of Minimum Bid Requirements

In the examples below, assume that the minimum bid increment is 10% of the current high bid.

*Example 1:* Three different bidders have the indicated high bids on packages involving two licenses, A and B.

A	B
10	
3	5

Case 1: The bidder holding the current high bid on license A wishes to submit a bid on the package {A,B}.

Step 1: The maximizing collection is {A,B}, and the associated amount is  $10 + 0.1 \cdot (10) = 11$ .

Step 2: Irrelevant. The minimum acceptable bid is 11.

Case 2: The bidder holding the current high bid on the package {A,B} wishes to raise its own bid on the package.

Step 1: The maximizing collection is {A,B}, and the associated amount is 10.

Step 2: Increase the amount determined in Step 1 by 1. The minimum acceptable bid is 11.

*Example 2:* Two different bidders have the indicated high bids on licenses A and B. Package {A,B} has no current high bid.

A	B
-	
3	5

Case 1: The bidder holding the current high bid on license A wishes to submit a bid on the package {A,B}.

Step 1: The maximizing collection is {A} and {B}, and the associated amount is  $3 + 5.5 = 8.5$ .

Step 2: Irrelevant. The minimum acceptable bid is 8.5.

Case 2: Some other bidder, not high on A or B, wishes to submit a bid on the package {A,B}.

Step 1: The maximizing collection is {A} and {B}, and the associated amount is  $3.3 + 5.5 = 8.8$ .

Step 2: Irrelevant. The minimum acceptable bid is 8.8.

*Example 3:* The same bidder has the indicated high bids on both licenses A and B. Package {A,B} has no current high bid.

A	B
-	
3	5

Case 1: The bidder holding the current high bids on licenses A and B wishes to submit a bid on the package {A,B}.

Step 1: The maximizing collection is {A} and {B}, and the associated amount is  $3 + 5 = 8$ .

Step 2: Increase the amount determined in Step 1 by 0.3. The minimum acceptable bid is 8.3.

## Minimum Offer Requirements

In order to determine the minimum acceptable offer a bidder is allowed to place on a target package, the steps below are carried out:

- (1) All currently-high bids and offers submitted by other bidders are raised (for this calculation only) by their minimum increments. The total-maximizing collection of non-overlapping bids and offers on subpackages of the target package is determined, and the associated total bids-and-offers amount is determined. (This is the minimum amount needed to “get in”.)

- (2) The total-maximizing collection of non-overlapping current bids and offers on subpackages of the complement of the target package is determined.
- (3) All bids and offers that are part of the current prospectively-winning collection are raised (for this calculation only) by their minimum bid increments. The smallest amount  $t$  with the following property is determined: If the amount determined in (1) is increased by  $t \bullet |\text{target package}|$  (i.e., by the product of  $t$  with the number of licenses in the target package), and each package bid in the collection determined in (2) is similarly increased (i.e., by the product of  $t$  with the number of licenses in that package), then the total-maximizing collection contains the target package.
- (4) The minimum acceptable offer on the target package is the amount determined in (1), plus  $t \bullet |\text{target package}|$ .

Several examples will be given below to illustrate these steps. The basic idea is simple, and is a generalization of the note at the end of the previous discussion of the threshold problem: In order for a collection of bids and offers to “top” a prospectively-winning package or collection of packages, someone must be willing to offer to pay, on a per-license basis, at least their “fair share” of the amount constituting the threshold which must be crossed. Only bidders prepared to pay that share or more are permitted to make offers.

It is worthy of note that, if only singleton packages are allowed,  $t$  will be zero. The lowest acceptable offer a bidder can place on a license (on a license on which it is currently not the high bidder) will be the currently-high bid plus the minimum bid increment. Since bids and offers get into the system only when they are high, and are both removed from the system when topped (in the singleton-packages-only case), bids and offers are equivalent in all respects. This procedure then reduces, for all practical purposes, to the SMR procedure.

### Examples of Minimum Offer Requirements

In all of the examples given below, it is assumed that the minimum bid increment is 10% of the current high bid.

*Example 1:* Three different bidders have the indicated high bids on licenses A and B.

A	B
20	
5	5

Case 1: The bidder on license A wishes to submit an offer on A.

Step 1: The “get-in” amount is 5, the bidder’s current bid on A.

Step 2: The total-maximizing complementary collection is {B} alone.

Step 3: The relevant value of  $t$  solves  $(5+t) + (5+t) = 22$ , i.e.,  $t = 6$ .



Step 4: The bidder must offer at least  $5 + (6 \bullet 1) = 11$ .

Note that at least one of the incumbent singleton bidders must be willing to offer at least this much in order for the two together to have any hope of topping the current package bid on  $\{A,B\}$ .

Case 2: Some other bidder wishes to submit an offer on A.

Step 1: The “get-in” amount is 5.5, the amount needed to top the bid on A.

Step 2: The total-maximizing complementary collection is  $\{B\}$  alone.

Step 3: The relevant value of  $t$  solves  $(5.5+t) + (5+t) = 22$ , i.e.,  $t = 5.75$ .

Step 4: The bidder must offer at least  $5.5 + (5.75 \bullet 1) = 11.25$ .

*Example 2:* Five different bidders have the indicated active high bids or offers. (The bid on  $\{A,B,C\}$  is prospectively winning.)

A	B	C
20		
10		
5	4	6

Case 1: The bidder on license C wishes to submit an offer on C.

Step 1: The “get-in” amount is 6, the bidder’s current bid on C.

Step 2: The total-maximizing complementary collection is  $\{A,B\}$  alone.

Step 3: The relevant value of  $t$  solves  $(10+2t) + (6+t) = 22$ , i.e.,  $t = 2$ .

Step 4: The bidder must offer at least  $6 + (2 \bullet 1) = 8$ .

Case 2: The bidder on  $\{A,B\}$  wants to make an offer on that package.

Step 1: The “get-in” amount is 10, the bidder’s current bid on  $\{A,B\}$ .

Step 2: The total-maximizing complementary collection is  $\{C\}$  alone.

Step 3: The relevant value of  $t$  solves  $(10+2t) + (6+t) = 22$ , i.e.,  $t = 2$ .

Step 4: The bidder must offer at least  $10 + (2 \bullet 2) = 14$ .

Case 3: Some other bidder wishes to submit an offer on  $\{B,C\}$ .

Step 1: The “get-in” amount is  $4.4 + 6.6 = 11$ .

Step 2: The total-maximizing complementary collection is  $\{A\}$  alone.

Step 3: The relevant value of  $t$  solves  $(5+t) + (11+2t) = 22$ , i.e.,  $t = 2$ .

Step 4: The bidder must offer at least  $11 + (2 \bullet 2) = 15$ .

*Example 3:* Five different bidders have the indicated active high bids or offers. (The bid on {A,B,C} is prospectively winning.) Note that this situation could occur (i.e., the prices displayed for {A} alone and {B} alone could sum to more than the {A,B} price) if the price shown for B were an offer.

A	B	C
20		
10		
5	8	6

Case 1: Some other bidder wishes to submit an offer on C.

Step 1: The “get-in” amount is 6.6.

Step 2: The total-maximizing complementary collection is {A} and {B}.

Step 3: The relevant value of  $t$  solves  $(5+t) + (8+t) + (6.6+t) = 22$ , i.e.,  $t = 0.8$ .

Step 4: The bidder must offer at least  $6.6 + (0.8 \bullet 1) = 7.4$ .

### Activity and eligibility

The threat of the auction ending puts pressure on bidders to remain “active” throughout the auction. However, the pace of the auction can be accelerated through the use of “activity rules”. For the “bids and offers” auction procedure, we suggest that the rules take the following form.

A bidder is deemed to be *active during a round* on a license if it

- (1) enters the round as the submitter of a bid or offer which is part of the initial prospectively-winning collection and contains that license,
- (2) enters the round with a currently-high bid on a package containing that license,
- (3) enters a bid meeting the minimum bid requirements on a package containing that license at some point during the round,
- (4) enters an offer on a package containing that license which becomes active (i.e., is publicly listed) at some point during the round, or
- (5) ends the round as the submitter of the package containing that license which is part of the prospectively-winning collection at the end of the round.

A bidder is deemed to be ***active at the moment*** on a license if it holds the currently-high bid or offer on a package containing that license, or has submitted (if the moment is during a quiet period) a bid or offer on a package containing that license.

In particular, the following do not contribute towards a bidder's activity:

- (1) entering an offer during the first quiet period that is topped before the end of that period, or
- (2) entering an offer during the second quiet period that is not part of the prospectively-winning collection at the end of the round

A bidder's ***activity level in a round*** is the sum of the FCC-determined activity "scores" assigned (before the auction begins) to the licenses on which the bidder is active at any time during that round. A bidder's ***activity level at the moment*** is defined similarly.

A bidder's ***eligibility level in a round*** is an FCC-determined multiple of the bidder's lowest activity level in all previous rounds

A bidder is ***eligible*** to submit a bid or offer on a package only if the submission of that bid or offer would leave that bidder's activity level at the moment at or below the bidder's eligibility level in the current round.

Throughout the auction, bidders are only allowed to submit bids or offers which they are eligible to submit.

### **Bidder Interaction**

If the "bids and offers" procedure were to be conducted via the Internet, what support would bidders need? At the beginning of each round, bidders must have access to the provisionally-winning collection of bids and offers, and to the currently-high bids on each (allowed) package. They must also know their eligibility level, and their activity level at the moment the round starts.

At the end of the first quiet period, they must be provided with the same information, as well as the currently-high offers on all packages.

During the open period, as quickly as possible, bidders must be informed when the provisionally-winning collection changes, and when new currently-high bids and offers appear. This information needn't be "pushed" at the bidders, as long as they can query the bidding system for its current status. (It is likely that most bidders will find themselves running one online connection to monitor updates while using a second connection to submit bids.)

Besides having the ability to ask for a full update of the auction status, bidders should also be able to query the system with respect to any package of licenses. The information that is returned should include

- (1) whether they are eligible to bid on the package,
- (2) the minimum acceptable bid they could submit,
- (3) the minimum acceptable offer they could submit,
- (4) the minimum amount they would need to bid or offer in order for this package to become immediately (during the open period) or potentially (during the quiet periods) part of a new provisionally-winning collection, and a listing of those packages of licenses that would go to the bidder in that new collection.
- (5) their “current” activity level for the round (i.e., what their activity level for the round would be if the round ended immediately; during a quiet period, this will not include the potential activity earned by offers submitted during that quiet period), and
- (6) their “updated current” activity level for the round if they submit an acceptable bid or offer on the package immediately.

Of course, a bidder must be aware that the responses to all but (5) can change in real time during the open period. During both quiet periods, responses will change only as the bidder enters bids.

### **Justification**

*Why break each round into three periods?* A primary reason for the first quiet period is to permit corporate bidding teams to work accordingly to a daily schedule. Any bidder submitting only bids during the first quiet period can determine its activity level in the round at the time the bids are submitted. Another reason is to avoid placing bidders in a race to be the first to submit an acceptable bid or offer. Note that, in the case where only single-license packages are allowed, if the durations of the open and second quiet periods are set to zero then the first quiet period becomes the equivalent of a round in the SMR procedure.

The reason for the open period is to allow time for a series of offers to fall into place. Each offer potentially reduces the minimum acceptable level for other offers needed to form a new prospectively-winning collection. Bidders willing to pay part of the cost of climbing over a threshold may not be able to submit the offers they are willing to make until other bidders have their offers on record.

The second quiet period is needed in order to avoid a “reversed” race, where several bidders seek to wait until the very end of the round, hoping that others will improve the offers on the table before acting themselves.

*Why would bidders wish to submit offers?* There are several reasons. The most obvious is in an attempt to solve the threshold problem. In addition, the temporary nature of an offer permits a bidder to attempt to bring other bidders into the effort to clear the threshold without making a unilateral “concession”. The literature on bargaining generally preaches the wisdom of making conditional offers (“I’ll do this if you’ll do that.”) rather than unconditional ones (“I’ll give you this for certain. Now, will you give me anything?”)

Offers also ease the exclusive-or problem, by allowing a bidder to shift its activity from one license or set of licenses to another between rounds. By submitting offers instead of bids, a bidder can avoid getting locked into one particular set of licenses, leaving the bidder free to shift eligibility around from one round to the next.

Finally, offers protect a bidder from getting locked into a position where, in later rounds when the bidder has shifted objectives, a bid might be resurrected by a combination of the bids of other bidders, i.e., offers help a bidder avoid bidding “against itself”.

*Why would bidders wish to submit bids?* Bids immediately contribute to a bidder’s activity in a round. Bidders with modest objectives—for example, bidders seeking only a single specific license—can bid at the beginning of a round and walk away until the next round starts. As well, a bidder unwilling to be the first to make an offer—perhaps because the bidder is truly unwilling or unable to bear the “lion’s share” of the burden in crossing some threshold—can maintain activity at a lower price level from round to round by bidding, and waiting for others to make the first offers.

## **Conclusion**

The “bids and offers” auction procedure is a direct extension of the SMR procedure already in use by the FCC. It deals with the exposure problem by allowing bids (and offers) on packages of licenses. It deals with the threshold and exclusive-or problems by allowing offers, which are really nothing more than temporary bids. It eliminates the need to deal with elective bid withdrawals, or “withdrawal cascades”. In addition, it holds some promise of speeding an auction to its conclusion by providing the bidders with multiple opportunities to submit bids in each round. For bidders wishing to submit only bids, the procedure is particularly transparent. With the appropriate access to exploratory feedback, it imposes a relatively light burden on bidders whose strategies involve the submission of offers.

For these reasons, we encourage the FCC to consider the use of this procedure in upcoming auctions. We also encourage fellow researchers to propose amendments if they seem

warranted, and we hope that experimenters will take the opportunity to examine the performance of this procedure in their laboratories.

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